

Neutrino Mixing and CP Violation in Matter

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Abstract. Within the framework of three lepton families I present a transparent analytical relationship between the neutrino mixing and CP-violating parameters in vacuum and those in matter. Such a model- and parametrization-independent result will be particularly useful to recast the fundamental lepton flavor mixing matrix from the future long-baseline neutrino experiments.

Today strong evidence, that neutrinos are massive and lepton flavors are mixed, has been accumulated from a variety of neutrino experiments. The mixing matrix of three different lepton families may in general consist of non-removable complex phases, leading to CP or T violation. Leptonic CP violation can manifest itself in neutrino oscillations. The best way to observe CP- and T-violating effects is to carry out the long-baseline neutrino experiments. In such experiments the earth-induced matter effects, which are likely to deform the neutrino oscillation behaviors in vacuum and fake the genuine CP-violating signals, must be taken into account. To single out the “true” theory of lepton mass generation and CP violation depends crucially upon how accurately the fundamental parameters of lepton flavor mixing can be measured and disentangled from the matter effects. It is therefore desirable to explore the most transparent analytical relationship between the genuine flavor mixing matrix and the matter-corrected one.

In this talk I present an exact and compact formula to describe the matter effect on lepton flavor mixing and CP violation within the framework of three lepton families [1]. The result is completely independent of the specific models of neutrino masses and the specific parametrizations of neutrino mixing. Therefore it will be particularly useful, in the long run, to recast the fundamental flavor mixing matrix from the precise measurements of neutrino oscillations in a variety of long-baseline neutrino experiments.

In vacuum the 3×3 lepton flavor mixing matrix V links the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$. If neutrinos are massive Dirac fermions, V can be parametrized in terms of three rotation angles and one CP-violating phase. If neutrinos are Majorana fermions, however, two additional CP-violating phases are in general needed to fully parametrize V . The strength of CP violation in neutrino oscillations, no matter whether neutrinos are of the Dirac or Majorana type, depends only upon a universal parameter \mathcal{J} [2]:

$$\text{Im} \left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^* \right) = \mathcal{J} \sum_{\gamma, k} \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} , \quad (1)$$

where (α, β, γ) and (i, j, k) run over (e, μ, τ) and $(1, 2, 3)$, respectively. In the specific models of fermion mass generation V can be derived from the mass matrices of charged leptons and neutrinos [3]. To test such theoretical models one has to compare their predictions for V with the experimental data of neutrino oscillations. The latter may in most cases be involved in the potential matter effects and must be carefully handled.

In the flavor basis where the charged lepton mass matrix M_l is diagonal (i.e., $M_l = \text{Diag}\{m_e, m_\mu, m_\tau\}$), the effective Hamiltonian responsible for neutrinos propagating in matter can be written as $\mathcal{H}_\nu = \Phi_\nu^m / (2E)$ with $\Phi_\nu^m = \Phi_\nu + \Phi_A$ and

$$\Phi_\nu = V \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} V^\dagger , \quad \Phi_A = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \quad (2)$$

Here m_i (for $i = 1, 2, 3$) denote neutrino masses, $A = 2\sqrt{2}G_F N_e E$ describes the charged-current contribution to the $\nu_e e^-$ forward scattering, N_e is the background density of electrons, and E stands for the neutrino beam energy. The neutral-current contributions, which are universal for ν_e , ν_μ and ν_τ neutrinos, lead only to an overall unobservable phase and have been neglected. One can diagonalize Φ_ν^m with a unitary transformation: $V^{m\dagger} \Phi_\nu^m V^m = \text{Diag}\{\lambda_1, \lambda_2, \lambda_3\}$, where λ_i denote the effective mass-squared eigenvalues of three neutrinos in matter. Explicitly we have

$$\begin{aligned} \lambda_1 &= m_1^2 + \frac{1}{3}x - \frac{1}{3}\sqrt{x^2 - 3y} \left[z + \sqrt{3(1 - z^2)} \right] , \\ \lambda_2 &= m_1^2 + \frac{1}{3}x - \frac{1}{3}\sqrt{x^2 - 3y} \left[z - \sqrt{3(1 - z^2)} \right] , \\ \lambda_3 &= m_1^2 + \frac{1}{3}x + \frac{2}{3}z\sqrt{x^2 - 3y} , \end{aligned} \quad (3)$$

where x , y and z are given by [4]

$$\begin{aligned} x &= \Delta m_{21}^2 + \Delta m_{31}^2 + A , \\ y &= \Delta m_{21}^2 \Delta m_{31}^2 + A \left[\Delta m_{21}^2 (1 - |V_{e2}|^2) + \Delta m_{31}^2 (1 - |V_{e3}|^2) \right] , \\ z &= \cos \left[\frac{1}{3} \arccos \frac{2x^3 - 9xy + 27A\Delta m_{21}^2 \Delta m_{31}^2 |V_{e1}|^2}{2(x^2 - 3y)^{3/2}} \right] \end{aligned} \quad (4)$$

with $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$. Note that the unitary matrix V^m is just the lepton flavor mixing matrix in matter. After a lengthy calculation we arrive at the elements of V^m as [1]

$$V_{\alpha i}^m = \frac{N_i}{D_i} V_{\alpha i} + \frac{A}{D_i} V_{ei} \left[(\lambda_i - m_j^2) V_{ek}^* V_{\alpha k} + (\lambda_i - m_k^2) V_{ej}^* V_{\alpha j} \right] , \quad (5)$$

where α runs over (e, μ, τ) and (i, j, k) over $(1, 2, 3)$ with $i \neq j \neq k$, and

$$\begin{aligned} N_i &= (\lambda_i - m_j^2) (\lambda_i - m_k^2) - A \left[(\lambda_i - m_j^2) |V_{ek}|^2 + (\lambda_i - m_k^2) |V_{ej}|^2 \right], \\ D_i^2 &= N_i^2 + A^2 |V_{ei}|^2 \left[(\lambda_i - m_j^2)^2 |V_{ek}|^2 + (\lambda_i - m_k^2)^2 |V_{ej}|^2 \right]. \end{aligned} \quad (6)$$

Obviously $A = 0$ leads to $V_{\alpha i}^m = V_{\alpha i}$. This exact and compact formula shows clearly how the flavor mixing matrix in vacuum is corrected by the matter effects. Instructive analytical approximations can be made for Eq. (5), once the hierarchy of neutrino masses is experimentally known or theoretically assumed [5].

If leptonic CP were an exact symmetry in vacuum, the determinant of the commutator $[\Phi_\nu, M_l^2]$ would vanish. As both M_l^2 and Φ_A are real diagonal matrices in the chosen flavor basis, we find that $[\Phi_\nu^m, M_l^2] = [\Phi_\nu, M_l^2]$ holds. Then one can derive the relationship between the CP-violating parameter \mathcal{J} and its counterpart in matter \mathcal{J}_m from the equality $\text{Det}[\Phi_\nu^m, M_l^2] = \text{Det}[\Phi_\nu, M_l^2]$. Following the calculations in Ref. [2], we arrive at

$$\mathcal{J}_m (\lambda_1 - \lambda_2) (\lambda_2 - \lambda_3) (\lambda_3 - \lambda_1) = \mathcal{J} \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2. \quad (7)$$

Such an elegant relation has already been observed in Ref. [6]. It indicates that the matter contamination to CP- and T-violating observables, which must be dependent upon \mathcal{J}_m , is in general unavoidable. Note that both \mathcal{J}_m and λ_i are complicated functions of the matter parameter A .

The results obtained above are valid for neutrinos interacting with matter. As for antineutrinos, the matter effects arise from the charged-current contribution to the “ $\bar{\nu}_e e^+$ forward scattering”. The corresponding formulas for antineutrino mixing can straightforwardly be obtained from Eqs. (5) and (7) through the replacements $V \Rightarrow V^*$ and $A \Rightarrow -A$.

The matter-corrected flavor mixing and CP-violating parameters can be determined from a variety of long-baseline neutrino experiments. We calculate the conversion probabilities of ν_α (or $\bar{\nu}_\alpha$) to ν_β (or $\bar{\nu}_\beta$) neutrinos in matter and obtain

$$\begin{aligned} P_m(\nu_\alpha \rightarrow \nu_\beta) &= -4 \sum_{i < j} [\text{Re}(V_{\alpha i}^m V_{\beta j}^m V_{\alpha j}^{m*} V_{\beta i}^{m*}) \sin^2 \Delta_{ij}] + 8 \mathcal{J}_m \prod_{i < j} \sin \Delta_{ij}, \\ P_m(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) &= -4 \sum_{i < j} [\text{Re}(\tilde{V}_{\alpha i}^m \tilde{V}_{\beta j}^m \tilde{V}_{\alpha j}^{m*} \tilde{V}_{\beta i}^{m*}) \sin^2 \tilde{\Delta}_{ij}] - 8 \tilde{\mathcal{J}}_m \prod_{i < j} \sin \tilde{\Delta}_{ij}, \end{aligned} \quad (8)$$

where (α, β) run over (e, μ) , (μ, τ) or (τ, e) ; $\tilde{V}_{\alpha i}(A) \equiv V_{\alpha i}(-A)$, $\tilde{\Delta}_{ij}(A) \equiv \Delta_{ij}(-A)$, and $\tilde{\mathcal{J}}_m(A) \equiv \mathcal{J}_m(-A)$; and $\Delta_{ij} \equiv 1.27(\lambda_i - \lambda_j)L/E$ with L the distance between the production and interaction points of ν_α (in unit of km) and E the neutrino beam energy (in unit of GeV). $P_m(\nu_\beta \rightarrow \nu_\alpha)$ and $P_m(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$ can be read off from Eq. (8) with the replacements $\mathcal{J}_m \Rightarrow -\mathcal{J}_m$ and $\tilde{\mathcal{J}}_m \Rightarrow -\tilde{\mathcal{J}}_m$, respectively.

Let me give a numerical illustration of the matter-induced corrections to the lepton flavor mixing matrix in vacuum. The elements of V^m , except the unknown

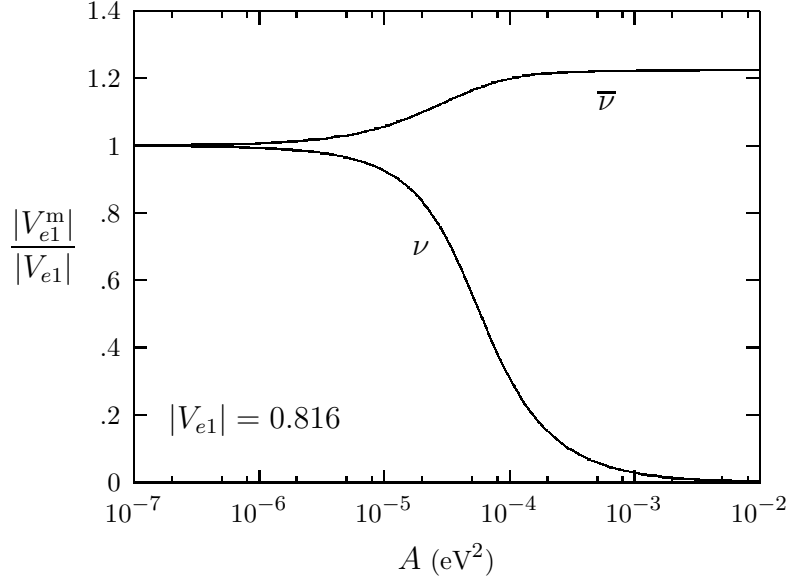


FIGURE 1. Illustrative plot for matter effects on $|V_{e1}|$ associated with neutrinos ($+A$) and antineutrinos ($-A$), where $\Delta m_{21}^2 = 5 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 3 \cdot 10^{-3} \text{ eV}^2$ have been input.

Majorana phases, can be completely determined by four rephasing-invariant quantities (e.g., four independent $|V_{\alpha i}^m|$ or three independent $|V_{\alpha i}^m|$ plus \mathcal{J}_m). As the solar and atmospheric neutrino oscillations in vacuum are essentially associated with the elements in the first row and the third column of V , it is favored to choose $|V_{e1}|$, $|V_{e2}|$, $|V_{\mu 3}|$ and \mathcal{J} as the four basic parameters. To be specific we take $|V_{e1}| = 0.816$, $|V_{e2}| = 0.571$, $|V_{\mu 3}| = 0.640$, and $\mathcal{J} = \pm 0.020$ for neutrinos and antineutrinos. Such a choice is consistent with the CHOOZ experiment [7], the large-angle MSW solution to the solar neutrino problem, and a nearly maximal mixing in the atmospheric neutrino oscillation [8]. The relevant neutrino mass-squared differences are typically taken to be $\Delta m_{21}^2 = 5 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 3 \cdot 10^{-3} \text{ eV}^2$. Then we compute the ratios $|V_{\alpha i}^m|/|V_{\alpha i}|$ and $\mathcal{J}_m/\mathcal{J}$ as functions of the matter parameter A in the range $10^{-7} \text{ eV}^2 \leq A \leq 10^{-2} \text{ eV}^2$. The numerical results are shown in Figs. 1 to 4.

We observe that matter effects can be significant for the elements in the first and the second columns of V , if $A \geq 10^{-5} \text{ eV}^2$. In comparison, the magnitudes of $|V_{e3}|$, $|V_{\mu 3}|$ and $|V_{\tau 3}|$ may be drastically enhanced or suppressed only for $A > 10^{-3} \text{ eV}^2$. The neutrinos are relatively more sensitive to the matter effects than the antineutrinos.

The magnitude of \mathcal{J}_m decreases, when the matter effect becomes significant (e.g., $A \geq 10^{-4} \text{ eV}^2$). However, this does not imply that the CP- or T-violating asymmetries in realistic long-baseline neutrino oscillations would be smaller than their values in vacuum. Large matter effects can significantly modify the frequencies of neutrino oscillations and thus enhance or suppress the genuine signals of CP or T

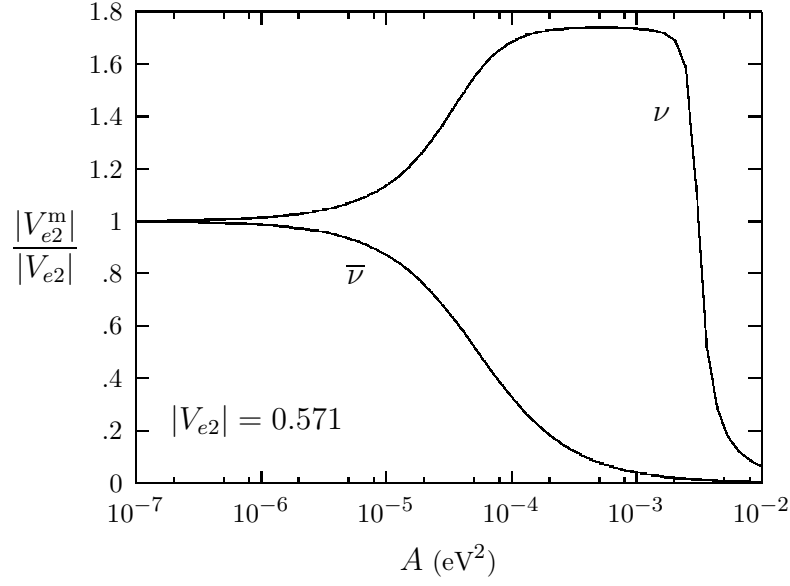


FIGURE 2. Illustrative plot for matter effects on $|V_{e2}|$ associated with neutrinos ($+A$) and antineutrinos ($-A$), where $\Delta m_{21}^2 = 5 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 3 \cdot 10^{-3} \text{ eV}^2$ have been input.

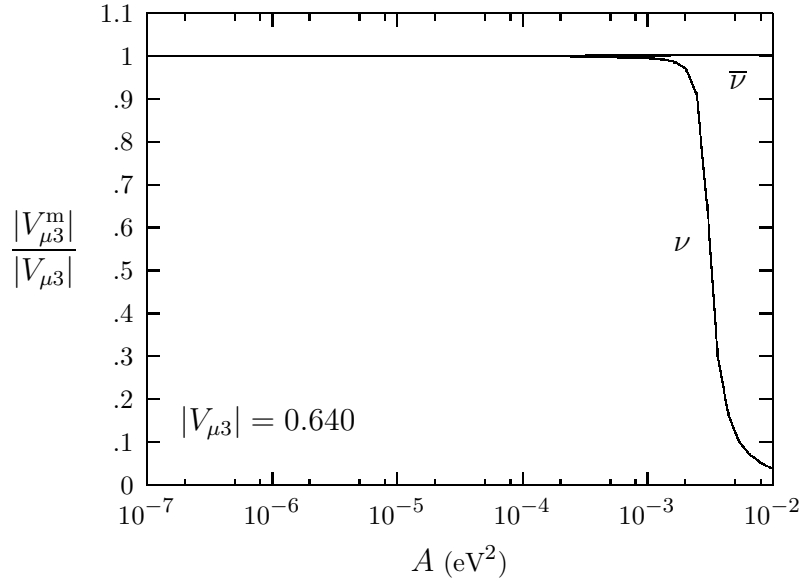


FIGURE 3. Illustrative plot for matter effects on $|V_{\mu3}|$ associated with neutrinos ($+A$) and antineutrinos ($-A$), where $\Delta m_{21}^2 = 5 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 3 \cdot 10^{-3} \text{ eV}^2$ have been input.

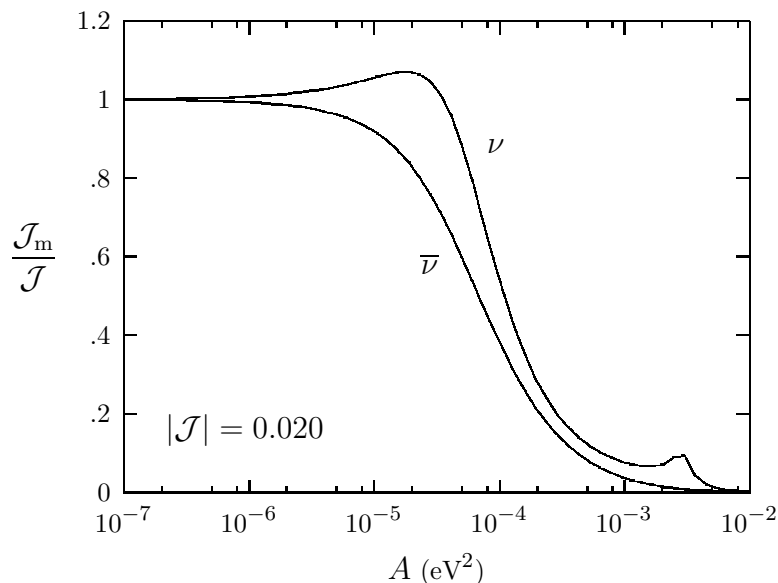


FIGURE 4. Illustrative plot for matter effects on \mathcal{J} associated with neutrinos ($+A$) and antineutrinos ($-A$), where $\Delta m_{21}^2 = 5 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 3 \cdot 10^{-3} \text{ eV}^2$ have been input.

violation.

If the earth-induced matter effects can well be controlled, it is possible to recast the fundamental flavor mixing matrix V from a variety of measurements of neutrino oscillations. Such a goal is expected to be reached in the neutrino factories [9,10].

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